

Lennard-Jones quark matter and massive quark stars

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ABSTRACT

Quark clustering could occur in cold quark matter because of the strong coupling between quarks at realistic baryon densities of compact stars. Although one may still not be able to calculate this conjectured matter from the first principles, the intercluster interaction might be analogized to the interaction between inert molecules. Cold quark matter would then crystallize in a solid state if the intercluster potential is deep enough to trap the clusters in the wells. We apply the Lennard-Jones potential to describe the intercluster potential and derive the equations of state, which are stiffer than those derived in conventional models (e.g. MIT bag model). If quark stars are composed of the Lennard-Jones matter, they could have high maximum masses ($>2 M_{\odot}$) as well as very low masses ($<10^{-3} M_{\odot}$). These features could be tested by observations.

Key words: dense matter – elementary particles – stars: neutron – pulsars: general.

1 INTRODUCTION

To understand the nature of pulsars, we need to know the state of cold quark matter, in which the dominant degree of freedom are quarks, and their Fermi energy is much larger than their thermal energy. However, this is a difficult task because of (i) the non-perturbative effect of the strong interaction between quarks at low energy scale and (ii) the many-body problem of vast assemblies of interacting particles.

On the one hand, some efforts have been made to understand the behaviour of quantum chromo-dynamics (QCD) at high density, among which a colour super-conductivity (CSC) state is currently focused on perturbative QCD as well as on QCD-based effective models (e.g. Alford et al. 2008). On the other hand, it is phenomenologically conjectured that astrophysical cold quark matter could be in a solid state (Xu 2003), since the strong interaction may render quarks grouped in clusters, and the ground state of realistic quark matter might not be that of the Fermi gas (see a recent discussion given by Xu 2009). If the residual interaction between quark clusters is stronger than their kinetic energy, each quark cluster could be trapped in the potential well and cold quark matter will be in a solid state. Solid quark stars still cannot be ruled out in both astrophysics and particle physics (Horvath 2005; Owen 2005). Additionally, there is evidence that the interaction between quarks is very strong in hot quark-gluon plasma (i.e. the strongly coupled quark-gluon plasma; Shuryak 2009) according to the recent achievements of relativistic heavy ion collision experiments. When the temperature goes down, it is reasonable to conjecture that

the interaction between quarks should be stronger than in the hot quark-gluon plasma.

Because of the difficulty in obtaining a realistic state equation of cold quark matter at a few nuclear densities, we try to apply some phenomenological models, which would have some implications on the properties of QCD at low energy scales if the astronomical observations can provide us with some limitations on such models. In our previous paper (Lai & Xu 2009), a polytropic quark star model has been suggested in order to establish a general framework in which theoretical quark star models could be tested by observations. This model can help us understand the observed masses of pulsars and the energy released during some extreme bursts; however, this is a phenomenological model and does not include the form of interaction between quarks. To calculate the interaction between quarks and to predict the state of matter for quark stars by QCD calculations are difficult tasks; however, it is still meaningful for us to consider some models to explore the properties of quarks at the low energy scale.

We can compare the interaction between quark clusters with the interaction between inert molecules. A single quark cluster inside a quark star is assumed to be colourless, just like each molecule in a bulk of inert gas is electric neutral. The interaction potential between two inert gas molecules is well described by the Lennard-Jones potential (Lennard-Jones 1924)

$$u(r) = 4U_0 \left[\left(\frac{r_0}{r} \right)^{12} - \left(\frac{r_0}{r} \right)^6 \right], \quad (1)$$

where U_0 is the depth of the potential and r_0 can be considered as the range of interaction. This form of potential has the property of short-distance repulsion and long-distance attraction. We assume that the interaction between the quark clusters in quark stars can

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also be described by the form of the Lennard-Jones potential.¹ If the intercluster potential is deep enough to trap the clusters in the potential wells, the quark matter would crystallize and form solid quark stars. Under such potential, we can get the equation of state for solid quark stars, where the pressure comes from both the intercluster potential and lattice vibrations. Because the chromo-interaction is stronger than the electromagnetic interaction that is responsible for the intermolecular forces, the values of parameters, U_0 and r_0 used in the cold quark matter, should be different from that in the inert gas, and should be determined in the context of quark stars.

The model of quark stars composed of the Lennard-Jones matter is much different from conventional models (e.g. MIT bag model) in which the ground state is of the Fermi gas. In the former case, the quark clusters are non-relativistic particles, whereas in the latter case quarks are relativistic particles. Consequently, the equations of state in these two kinds of models are different, and we find that the Lennard-Jones model has stiffer equations of state, which lead to higher maximum masses for quark stars. Certainly, quark stars can have a very low mass ($<10^{-3} M_\odot$) due to self colour interaction. On the other hand, we find that under some reasonable values of parameters, a quark star could also be very massive ($>2 M_\odot$).

This Letter is arranged as follows. The details of lattice thermodynamics are listed in Section 2. The forms of equations of state are given in Section 3, including the comparison with the MIT bag model, and we show the corresponding mass–radius curves in Section 4. We make conclusions and discussions in Section 5.

2 LATTICE THERMODYNAMICS

For an intercluster potential which is deep enough to trap the clusters in the potential wells, the quark matter would crystallize to get lower energy and form solid quark stars. In this section, we use the results in classical solid physics to discuss the properties of crystallized cold quark matter.

2.1 The intercluster potential

Like the inert gas, the interaction potential u between two quark clusters as the function of their distance r is described by the Lennard-Jones potential (equation 1). Let us consider a system containing N clusters with volume V . The total interaction potential is

$$U = \frac{1}{2} \sum_i \sum_{j \neq i} u(r_{ij}), \quad (2)$$

and if we ignore the surface tension, we get

$$U = \frac{N}{2} \sum_{j \neq i} u(r_{ij}) = \frac{N}{2} \sum_{j \neq i} \left\{ 4U_0 \left[\left(\frac{r_0}{r_{ij}} \right)^{12} - \left(\frac{r_0}{r_{ij}} \right)^6 \right] \right\}. \quad (3)$$

The lattice structure of cold quark matter is unknown, and we adopt the simple-cubic structure. The cold quark matter may have other

kinds of structures, but that will not make much difference at least quantitatively. If the nearest distance between two quark clusters is R , then the total interaction potential of N quark clusters is

$$U(R) = 2NU_0 \left[A_{12} \left(\frac{r_0}{R} \right)^{12} - A_6 \left(\frac{r_0}{R} \right)^6 \right], \quad (4)$$

where $A_{12} = 6.2$ and $A_6 = 8.4$. In the simple-cubic structure, the number density of clusters n is

$$n = R^{-3}, \quad (5)$$

so

$$U(R) = 2NU_0 (A_{12}r_0^{12}n^4 - A_6r_0^6n^2), \quad (6)$$

and the potential energy density is

$$\epsilon_p = 2U_0 (A_{12}r_0^{12}n^5 - A_6r_0^6n^3). \quad (7)$$

2.2 Lattice vibrations

Consider a system of volume V containing N quark clusters. Each quark cluster in the crystal lattice undergoes a three-dimensional vibration about its lattice site. Performing a normal-mode analysis in which the vibrations of the lattice are decomposed into $3N$ independent normal modes of vibrations, the total lattice vibration is a superposition of these $3N$ decoupled vibrations.

For cold quark matter, the thermal vibration can be neglected compared to the zero-point energy of phonon, so the average energy of an individual mode of vibration with frequency ω_j is

$$E_j = \frac{1}{2} \hbar \omega_j. \quad (8)$$

Using the Debye approximation, at low temperature, the thermodynamic properties of crystal lattice are mainly determined by the long-wavelength sound waves. The propagation of the wave can be decomposed into one longitude mode with velocity v_{\parallel} and two transverse modes with velocity v_{\perp} . The total velocity v has the relation

$$\frac{1}{v^3} = \frac{1}{3} \left(\frac{1}{v_{\parallel}^3} + \frac{2}{v_{\perp}^3} \right), \quad (9)$$

and the total energy of the $3N$ vibrations is

$$\bar{E} = \int_0^{\omega_m} \frac{1}{2} \hbar \omega f(\omega) d\omega, \quad (10)$$

where $f(\omega) d\omega$ is the number of modes in the interval from ω to $\omega + d\omega$, and ω_m is the maximum frequency related to the non-continuous structure of the solid, and is determined by

$$\omega_m = v(6\pi^2 n)^{1/3}. \quad (11)$$

Under the condition of zero temperature, the integration can be done and the total energy of the crystal vibrations becomes

$$\bar{E} = \frac{9V}{8} (6\pi^2)^{\frac{1}{3}} \hbar v n^{\frac{4}{3}}, \quad (12)$$

so the energy density of the lattice vibration is

$$\epsilon_L = \frac{9}{8} (6\pi^2)^{\frac{1}{3}} \hbar v n^{\frac{4}{3}}. \quad (13)$$

¹The interaction between nuclei can be described by the $\sigma - \omega$ model (Walecka 1974), which is also characterized by the short-distance repulsion and long-distance attraction. Recently, the nucleon–nucleon potential has been studied by lattice QCD simulations (Ishii, Aoki & Hatsuda 2007), and they also derive a strong repulsive core at short distances. The interaction between quark clusters in cold quark matter could also have long-distance attraction and short-distance repulsion if scale and vector mesons contribute there. We note that this short-distance repulsion is essential to reproduce a stiff equation of state in our model.

3 EQUATION OF STATE FOR QUARK STARS

3.1 Quark stars composed of the Lennard-Jones matter

The total energy density for cold quark matter is

$$\begin{aligned}\epsilon_q &= \epsilon_p + \epsilon_L + nm_c c^2 \\ &= 2U_0 (A_{12} r_0^{12} n^5 - A_6 r_0^6 n^3) \\ &\quad + \frac{9}{8} (6\pi^2)^{\frac{1}{3}} \hbar v n^{\frac{4}{3}} + nm_c c^2,\end{aligned}\quad (14)$$

where m_c is the mass of each quark cluster. The pressure can be derived as

$$\begin{aligned}P_q &= n^2 \frac{d(\epsilon_q/n)}{dn} \\ &= 4U_0 (2A_{12} r_0^{12} n^5 - A_6 r_0^6 n^3) + \frac{3}{8} (6\pi^2)^{\frac{1}{3}} \hbar v n^{\frac{4}{3}}.\end{aligned}\quad (15)$$

Apart from quarks, there are electrons in quark matter. In the MIT bag model, the number of electrons per baryon N_e/A is found for different strange quarks mass m_s and coupling constant α_s (Farhi & Jaffe 1984). In their results, when $\alpha_s = 0.3$, N_e/A is less than 10^{-4} ; a larger α_s means a smaller N_e/A at a fixed m_s , because the interaction between quarks will lead to more strange quarks and consequently less electrons. In our model, we also consider the strong interaction between quarks as well as between quark clusters, and consequently the required number of electrons per baryon to guarantee the neutrality should be very small. Although at the present stage we have not got the exact value for the number density of electrons, we assume that N_e/A is less than 10^{-4} . We find that even at this value, the pressure of the degenerate electrons is negligible compared to the pressure of quarks. Therefore, we neglect the contribution of electrons to the equation of state. Then, the equation of state for quark stars is

$$P = P_q, \quad (16)$$

$$\rho = \epsilon_q/c^2. \quad (17)$$

3.2 Parameters

Up to now, there are a couple of parameters in the equation of state, and in this section we will show how to determine them.

(1) *The long-wavelength sound speed v for lattice vibration.* For the extremely relativistic systems, the sound velocity is $1/\sqrt{3}$, and in general it will be less than $1/\sqrt{3}$. We find that the equation of state does not change much when v goes from the velocity of light c to $10^{-5}c$, so in our calculations we set $v = c/3$.

(2) *The mass of quarks.* Quark stars are composed entirely of deconfined light quarks (up, down and strange quarks), the so-called strange stars. Because the deconfined phase and the chiral-restoration phase transitions might not occur simultaneously in the QCD phase diagram, we give each quark a constituent mass and assume that it is one-third of the nuclear mass.

(3) *The number of quarks in one cluster N_q .* Quarks are fermions and have three flavour (up, down and strange) degrees of freedom and three colour degrees of freedom. Pauli's exclusion principle tells us that if in the inner space quarks are exchange-asymmetric, they are exchange-symmetric in position space and they have a tendency to condensate in position space. We therefore conjecture the

existence of quark clusters in quark matter and leave the number of quarks in one cluster, N_q , as a free parameter. An 18-quark cluster, called quark-alpha (Michel 1988), could be completely asymmetric in spin, flavour and colour space, so in our calculation we set $N_q = 18$, and we also set $N_q = 3$. On the other hand, it has been conjectured that strongly interacting matter at high densities and low temperatures might be in a 'quarkyonic' state, which also contains three quarks in one cluster, and is characterized by chiral symmetry and confinement (McLerran & Pisarski 2007; Blaschke, Sandin & Klahn 2008). However, in our model, the state is characterized by chiral symmetry breaking and deconfinement.

(4) *The depth of the potential U_0 .* Given the density of quark matter ρ and the mass of each individual quark from Heisenberg's uncertainty relation, we can approximate the kinetic energy of one cluster as

$$E_k \sim 1 \text{ MeV} \left(\frac{\rho}{\rho_0} \right)^{\frac{2}{3}} \left(\frac{N_q}{18} \right)^{-\frac{5}{3}}, \quad (18)$$

where ρ_0 is the nuclear matter density. To get the quarks trapped in the potential wells to form a lattice structure, U_0 should be larger than the kinetic energy of quarks. Because of the strong interaction between quarks, we adopt $U_0 = 50$ and 100 MeV to do the calculations.

(5) *The range of the action r_0 .* Since a quark star could be bound not only by gravity but also by strong interaction due to the confinement between quarks, the number density of quarks on a quark star surface ρ_s is non-zero. For a given ρ_s , we can get r_0 at the surface where the pressure is zero. We choose ρ_s as two times nuclear matter densities and get the value of r_0 accordingly, which is found in the range from about 1 to 3 fm.

When U_0 and r_0 are given, the intercluster potential equation (1) is fixed. One should note that it describes the interaction between only two clusters; if we consider other clusters' influences, a cluster will always be in the minimal potential state. When the cluster deviates from the equilibrium position, it will be pulled back due to the stronger repulsion from one side, just as the case of a chain of springs.

3.3 Comparison with the MIT bag model

In the MIT bag model, quark matter is composed of massless up and down quarks, massive strange quarks and a few electrons. Quarks are combined together by an extra pressure, denoted by the bag constant B . For the comparison, we apply the formulae given by Alcock, Farhi & Olinto (1986) to calculate the equation of state, with strange quark mass $m_s = 100$ MeV, the strong coupling constant $\alpha_s = 0.3$ and the bag constant $B = 60$ MeV/fm⁻³ (e.g. Zdunik 2000). The comparison of equation of state in our model and in the MIT bag model is shown in Fig. 1.

In our model, quarks are grouped in clusters and these clusters are non-relativistic particles. If the intercluster potential can be described as the Lennard-Jones form, the equation of state can be very stiff, because at a small intercluster distance (i.e. the number density is large enough), there is a very strong repulsion. However, in the MIT bag model quarks are relativistic particles (at least for up and down quarks). For a relativistic system, the pressure is proportional to the energy density, so it cannot have stiff equation of state.

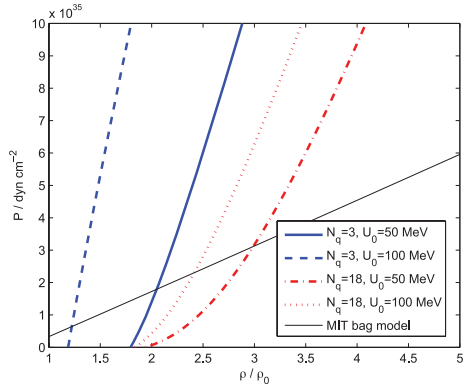


Figure 1. Comparison of the equations of state with $N_q = 3$, including $U_0 = 50$ MeV (blue solid lines) and $U_0 = 100$ MeV (blue dashed lines), and the corresponding case $N_q = 18$ with $U_0 = 50$ MeV (red dash-dotted lines) and $U_0 = 100$ MeV (red dotted lines), and that derived in the MIT bag model with the mass of strange quark $m_s = 100$ MeV and the strong coupling constant $\alpha_s = 0.3$ and bag constant $B = 60$ MeV/fm $^{-3}$ (thin lines), for a given surface density $\rho_s = 2\rho_0$. Here and in the following figures, ρ_0 is the nuclear saturation density.

3.4 The speed of sound

The adiabatic sound speed is defined as

$$c_s = \sqrt{dP/d\rho}. \quad (19)$$

If we use the equation of state in our model, the speed of sound will exceed the speed of light not far away from the surface of a quark star. It seems to contradict the relativity that signals cannot propagate faster than light.

The possibility of the speed of sound exceeding the speed of light in ultradense matter has been discussed previously (Bludman & Ruderman 1968), and was considered a consequence of using a classical potential (i.e. a kind of action at a distance). The physical reasons of apparent superluminal speed of sound have also been analysed (Caporaso & Brecher 1979). The authors argued that the adiabatic sound speed can exceed the speed of light, yet signals propagate at a speed less than c .

One reason is that the $P(\rho)$ relation arises from a static calculation, ignoring the dynamics of the medium. The notion that c_s is a signal propagation speed is a carry-over from Newtonian hydrodynamics, in which one assumes infinite interaction speed but finite temperature, so the static and dynamic calculations give the same result. On the other hand, if one assumes finite interaction speed and zero temperature, the adiabatic sound speed is not a dynamically meaningful speed, but only a measure of the local stiffness. Another reason is that a lattice does not have an infinite range of allowed frequencies of vibration, but a signal should contain components at all frequencies. Therefore, the adiabatic sound speed is not capable of giving the velocity of propagation of disturbances.

In our model, although we have not made it explicit how the particles interact with each other, we may assume that the interaction is mediated by some particles with non-zero masses, and the interaction does not propagate instantaneously. We have also used the low frequency approximation to calculate the lattice energy. Therefore, we could conclude that in our model the signal cannot propagate faster than light.

Whether the equation of state of cold quark matter can be so stiff that the adiabatic speed of sound is larger than c could still be an open question. However, in our present Letter, we do not limit the

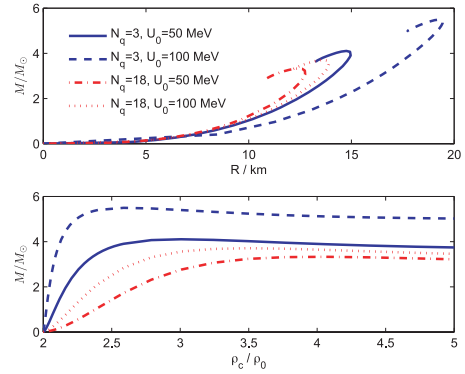


Figure 2. The mass–radius and mass–central density (rest-mass energy density) curves, in the case $N_q = 3$, including $U_0 = 50$ MeV (blue solid lines) and $U_0 = 100$ MeV (blue dashed lines), and the corresponding case $N_q = 18$ with $U_0 = 50$ MeV (red dash-dotted lines) and $U_0 = 100$ MeV (red dotted lines), for a given surface density $\rho_s = 2\rho_0$.

adiabatic sound speed and only treat it as a measurement of the stiffness of the equation of state.

4 MASSES AND RADII

From the equations of state, we can get the mass–radius and mass–central density curves (the central density only includes the rest-mass energy density), as shown in Fig. 2.

Because of stiffer equations of state, which we have discussed in Section 3, the maximum masses of quark stars in our model could be higher. In Fig. 2, we can see that (i) a deeper potential well U_0 means a higher maximum mass and (ii) if there are more quarks in a quark cluster, the maximum mass of a quark star will be lower.

A stiffer equation of state leading to a higher maximum mass could have very important astrophysical implications. Although we could still obtain high maximum masses under the MIT bag model by choosing suitable parameters (Zdunik et al. 2000), with a more realistic equation of state in the density-dependent quark mass model (e.g. Dey et al. 1998) it is very difficult to reach a high enough maximum stellar mass, which was considered as possible negative evidence for quark stars (Cottam, Paerels & Mendez 2002). Some recent observations have indicated some massive ($\sim 2M_\odot$) pulsars (e.g. Freire et al. 2008); however, because of the uncertain inclination of the binary systems, we are still not sure about the real mass. Though we have not definitely detected any pulsar whose mass is higher than $2M_\odot$ up to now, the Lennard-Jones quark star model could be supported if massive pulsars ($> 2M_\odot$) are discovered in the future. Moreover, a high maximum mass for quark stars might be helpful for us to understand the mass-distribution of stellar-mass black holes (Bailyn et al. 1998), since a compact star with a high mass (e.g. $\sim 5M_\odot$) could still be stable in our model presented.

5 CONCLUSIONS AND DISCUSSIONS

In cold quark matter at realistic baryon densities of compact stars (with an average value of $\sim 2-3\rho_0$), the interaction between quarks is so strong that they would condensate in position space to form quark clusters. Like classical solids, if the intercluster potential is deep enough to trap the clusters in the potential wells, the quark matter would crystallize and form solid quark stars. This picture of quark stars is different from the one in which quarks form Cooper pairs and quark stars are consequently colour super-conductive.

In this Letter, we argue that quarks in quark stars are grouped in clusters and the quark clusters form simple-cubic structure. We applied the Lennard-Jones potential to describe the interaction potential between quark clusters. The parameters such as the depth of potential U_0 (50 and 100 MeV) and the range of interaction r_0 (about 1 to 3 fm) are given by the physical context of quark stars. Under such equations of state, the masses and radii of quark stars are derived, and we find that the mass of a quark star can be higher than $2 M_{\odot}$.

It is surely interesting to experimentally or observationally distinguish between our solid quark star model and other models for quark stars, e.g. the CSC state. Starquakes could naturally occur in solid quark stars and the observations of pulsar glitches and soft γ -ray repeater (SGR) giant flares could qualitatively be reproduced when the solid matter breaks (Zhou et al. 2004; Xu, Tao & Yang 2006); moreover, the post-glitch recoveries in the solid quark star model and the CSC model would be different. Additionally, because the solid quark star model depends on quark clustering, the interaction behaviours between quarks could be tested in sQGP (strongly coupled quark-gluon plasma; see Shuryak 2009) by the Large Hadron Collider (LHC) and/or Facility for Antiproton and Ion Research (FAIR) experiments.

The study of compact stars involves two kinds of challenges: particle physics and many-body physics. Nevertheless, if we know about the properties of compact stars from observations, we can get information on the elementary physics. Take the model we discussed in this Letter as an example. If we get the masses and radii of some pulsars from accurate enough observations, we can put limits on the parameters such as potential well depth U_0 , interaction range r_0 and the number of quarks that condensate in position space to form a cluster, which could help us to explore the strong interaction between quarks. Although the state of cold quark matter at a few nuclear densities is still an unsolved problem in low-energy QCD, it would be helpful for us to use pulsars as idea laboratories to study the nature of the strong interaction.

In general, stars are equilibrium bodies with pressure against gravity. The thermal and radiation pressure dominates in main sequence stars, while degenerate pressure of Fermions, originated from Pauli's principle, dominates in Fermion stars (e.g. white dwarfs). For solid quark stars in the models presented in this Letter, the pressure is related to the increase in both potential and lattice vibration energies as the stellar quark matter contracts. The degenerate pressure might be negligible there.

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